Dimension and Boolean dimension of partial orders

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(abstract)

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The main goal of this project is to attack and resolve major research challenges in the theory of partially ordered sets, widely known as posets. This includes both algorithmic and combinatorial problems. Such a goal will most likely require building a modern structure theory for posets. While structural and extremal graph theory have flourished over last 30 years, combinatorial theory for posets is only now in a rapidly emerging phase. This project is rather problem-oriented, and we distinguish four directions of intended research.

Structural: The most important measure of a poset complexity is dimension. What structures are forced to appear in posets with large dimension? Is it true that large dimension in a planar poset must be witnessed by a large canonical structure, the standard examples?

Extremal: The maximum chromatic number of n-vertex triangle-free graphs is well-known to be Theta(sqrt{n/log(n)). The analogous statement for posets would be to determine the maximum dimension of n-element posets without standard example of size 3. For now, we only know that it is o(n). This upper bound shall be substantially improved.

Sparsity: The quest to find the right notion of sparse classes of graphs is a rapidly growing field in theoretical computer science. The notions of bounded expansion and nowhere denseness uncovered deep links between combinatorial, algorithmic, and logical view points. These concepts tie in unexpectedly with the combinatorics of posets. The property of a graph class being nowhere dense can be captured by looking at the dimension of posets whose cover graphs are in the class. Can we do the same for classes of bounded expansion?

Logic and computation: Focusing on the encoding aspect, Nešetřil and Pudlák in 1989, proposed a more expressive version of poset dimension, the Boolean dimension. They posed a beautiful, and still open, problem: Is the Boolean dimension of planar posets bounded? If true, this would imply a breakthrough result on the theoretical computer science side---a very efficient labelling scheme for the reachability problem in planar digraphs.