Homework 2

Due to October 22, 23:59.

Exercise 1. Let A be an antichain in P and $\text{width}(P - A) \leqslant w$. Show that

$$\dim(P) \leqslant 2w + 1.$$

Exercise 2. Let C_1, C_2 be totally incomparable chains of P, i.e., for all $x \in C_1$ and $y \in C_2$ we have $x \| y$ in P. Show that

$$\dim(P) \leqslant 2 + \dim(P \smallsetminus (C_1 \cup C_2)).$$

Exercise 3. Let P and Q be orders that both have a global minimum $\mathbf{0}$ and a global maximum $\mathbf{1}$ (and $\mathbf{0} \neq \mathbf{1}$ in both orders). Show that

$$\dim(P \times Q) = \dim(P) + \dim(Q).$$

Exercise 4. A graph is *chordal* if it does not contain an induced cycle of length more than 3. Prove that the set of prefixes of strings induced by perfect elemination orders of a chordal graph form an antimatroid.

It might be useful to know that the following is equivalent:

(i) G is chordal;
(ii) G is an intersection graph of subtrees of a tree, i.e. there is a tree T and a family of subrees $(T_v)_{v \in V(G)}$ of T such that for every two vertices u and v in G, u and v are adjacent if and only if $T_u \cap T_v$ are non-empty (share a vertex).

Exercise 5. Show that the Boolean lattice $\mathcal{B}_n$ with $n \geqslant 2$ has an orthogonal pair of chain partitions. (This was partially covered in Lecture 4.)

Exercise 6. How many antichains does the product $\mathbf{k} \times \mathbf{l}$ of two chains have?