Homework 13

Due to January 31, 23:59.

Please submit solutions of the following. To be graded.

Exercise 1. Try to find good time bounds for computing

a) The Ferrer's shape Fer(P) of a poset P.

b) A maximum k-chain in P.

c) A maximum ℓ-antichain in P.

Exercise 2. Find an analog of the Greene-Kleitman theory for directed acyclic graphs. After some thought you can look up a paper on this topic: Stefan Felsner, Orthogonal Structures in Directed Graphs, Journal of Combinatorial Theory (B) 57 (1993), 309-321.
Remark: This was made possible by Lemma (a) from Lecture 27.

Exercise 3. Given a 2-dimensional poset P, find good time bounds for computing

a) The skeleton of P.

b) The Ferrer's shape Fer(P) of P.

c) A maximum k-chain in P.

Exercise 4. This refers to Lecture 28, so P is 2-dimensional. Show that the k-chain $\mathcal{C}$ of X obtained from a maximum $(k-1)$-chain $\mathcal{C}'$ of $S(X)$ is maximum without refering to ℓ-antichains and orthogonal pairs.
Hint: Define a mapping from k chains in X to $k-1$ chains in $S(X)$.

Exercise 5. Visit the pages of Xavier Viennot (www.viennot.org and www.xavierviennot.org), stray around on these pages and tell us about findings you enjoyed.

Exercise 6. Within Lecture 29 we have seen that Algorithm has a strategy in on-line partition game on up-growing orders of width at most w to use at most $\binom{w+1}{2}$ chains. Show that this is best possible (so devise a strategy for Presenter).

Exercise 7. Find the value of the on-line antichain partition game on interval orders of height at most h presented with representation.

Exercise 8. Find the value of the on-line chain partition game on semi-orders of width at most w (presented without representation).