Shift graphs and others

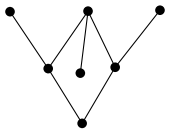
Exercise 1. Let D be a directed graph. An *arc-coloring* of D is an assignment of colors to arcs such that consecutive arcs obtain different colors. *Arc chromatic number* of D, denoted by $A(D)$ is the least integer k such that D has an arc-coloring with k colors. Show that

$$A(D) \geqslant \log \chi(D).$$

Exercise 2. Let n, k be integers with $n > 1$ and $k \geqslant 2$. A *generalized shift graph* G_n^k is the graph with the vertex set $V(G_n^k) = \binom{[n]}{k}$ an two k-tuples $\{x_1 < \cdots < x_k\}$, $\{y_1 < \cdots < y_k\}$ are adjacent if $x_2 = y_1, \ldots, x_k = y_{k-1}$ lub $y_2 = x_1, \ldots, y_k = x_{k-1}$. Show that for every fixed k the family of graphs $\{G_n^k\}_{n>1}$ has unbounded chromatic number

Hint: Use the previous exercise.

Exercise 3. Use critical pairs to show that the following poset is 3-dimensional.



Exercise 4. Show that posets with the same cover-graph need not have the same dimension. How large can the difference be?