

## Schnyder woods

A *3-orientation* of a triangulation is an orientation of inner edges such that each inner vertex has exactly 3 outgoing edges. The outer edges remain undirected.

*cw* - clockwise

*ccw* - counterclockwise

**Exercise 1.** Let  $\mathcal{T}$  be a 3-orientation of a triangulation  $G$ . Show that

- (i) If  $C$  is a directed cycle in  $\mathcal{T}$  such that for every edge  $e$  incident with one vertex in  $C$  and contained in the region bounded by  $C$ , the edge  $e$  is oriented towards a vertex in  $C$ , then  $C$  is a triangle.
- (ii) If  $\mathcal{T}$  contains a cw (ccw) cycle  $C$  then it also contains a cw (ccw) triangle contained in the region bounded by  $C$ .

**Exercise 2.** Prove that every 3-orientation  $\mathcal{T}$  of a triangulation  $G$  gives a unique Schnyder wood (up to cyclic shifts of colors RGB) such that the orientation of edges in the  $\mathcal{T}$  agrees with the orientation of edges in the Schnyder wood. Give an algorithm producing such a Schnyder wood.