Project Description – Project Proposals

Gwenaël Joret, Brussels; Piotr Micek, Kraków; Torsten Ueckerdt, Karlsruhe

Product structure of graphs

Project Description

Sections 1-3 must not exceed 17 pages in total.

1 Starting Point

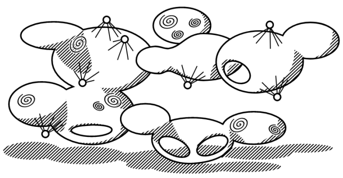
The need to deal with huge and complex networks has led to seminal results in graph theory that mark its coming of age as a mathematical field. A key result is Szemerédi's Regularity Lemma, which shows that every dense graph can be decomposed into a well-structured part and a random-like part, up to a small error term [55]. This work was largely responsible for Szemerédi winning the Abel prize, and led in part to the work of Tao for which he won a Fields medal.

While Szemerédi's Regularity Lemma describes the structure of dense graphs, at the opposite end of the spectrum the celebrated Robertson-Seymour theory describes the structure of graphs excluding a fixed graph as a minor. These are key examples of sparse graphs, graphs with a number of edges linear in the number of vertices. A *minor* of a graph is any graph obtained by removing vertices, removing edges, and contracting edges (in any order). Prime examples of graph classes closed under taking minors are planar graphs, and more generally graphs that can be drawn on a fixed surface without edge crossings.

Graph minor theory takes its roots in Kuratowski's theorem characterizing planar graphs as the graphs excluding K_5 and $K_{3,3}$ as minors. Starting in the late 1970's, Robertson and Seymour developed a rich theory in a series of 23 papers, for which they were awarded the Fulkerson Prize in 2006. This series culminated in the proof of the *Graph Minor Theorem* [52] showing that a Kuratowski-type theorem exists for every graph class that is closed under taking minors: Every minor-closed class of graphs can be characterized by excluding some *finite* set of graphs as minors. In his textbook on graph theory, Diestel [18] described this theorem as "one of the deepest theorems that mathematics has to offer".

A surface.[1]

The heart of Robertson and Seymour's proof of the Graph Minor Theorem is an approximate structure theorem for graphs excluding a fixed graph as a minor, the Graph Minor Structure Theorem [51]. This theorem shows that such graphs have a fundamentally 2-dimensional structure that is not too far from planar graphs. The building blocks are graphs embedded in a surface of bounded genus to which some controlled noise is added.



Structure of graphs excluding a minor.[1]

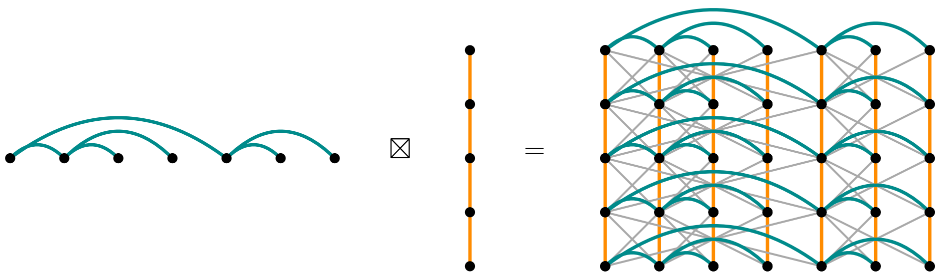
The Graph Minor Structure Theorem had a tremendous impact in graph theory, extending well beyond its role in the proof of the Graph Minor Theorem. However, the theorem has a drawback which is perhaps not obvious, namely, *planar graphs appear as a building block*. Indeed, sometimes the real difficulty comes from planar graphs.

[1] Image credit: Felix Reidl, used with permission.

For this reason, one would like to express planar graphs in terms of graphs with a simpler structure, in a way that proves useful for structural and algorithmic applications. This has been done very recently. Building on pioneering works by Pilipczuk and Siebertz [46], together with our coauthors we proved the following 'product structure theorem' [24]. This result was first accepted at the conference FOCS, which along with STOC is the most selective conference in theoretical computer science, and then in the *Journal of the ACM*, the most prestigious journal in theoretical computer science.

Theorem 1 (Product Structure Theorem)**.** *Every planar graph is a subgraph of the strong product of a graph of treewidth* 8 *and a path.*

The *strong product* of graphs G and H is the graph with vertex set $V(G) \times V(H)$ where (u, x) is adjacent to (v, y) if $u = v$ and xy is an edge, or $x = y$ and uv is an edge, or uv and xy are both edges. Here is an illustration of the strong product of a tree and a path:



Treewidth is a measure of how similar a graph is to a tree (the lower the better). It is a fundamental invariant in graph minor theory. Graphs of bounded treewidth have a simple tree-like structure. Thus, one can think of the above theorem as a recipe for extending results that are known to hold for bounded treewidth graphs to planar graphs. Considerable success has already been obtained in this way since the theorem appeared, leading to proofs of some decades-old conjectures. We give here a very brief account, see the next section for more details.

1. We showed that the 'queue-number' of planar graphs is bounded by an absolute constant, confirming a much-studied conjecture of Heath, Leighton, and Rosenberg from 1992. The resulting paper [24] was accepted at the conference FOCS, and in the *Journal of the ACM*.
2. We obtained a short proof of a well-known conjecture of Alon, Grytczuk, Hałuszczak, and Riordan from 2002, stating that planar graphs can be colored using a bounded number of colors in such a way that no path is 'repetitively colored'. The paper [19] was published in the new journal *Advances in Combinatorics* launched by Tim Gowers and Dan Král in 2019.
3. We obtained asymptotically optimal bounds for the 'adjacency labeling problem' for planar graphs. The resulting paper [23] was accepted at the conference FOCS, and in the *Journal of the ACM*.
4. We found (essentially) optimal constructions of universal graphs for planar graphs, which are graphs containing all n-vertex planar graphs as subgraphs. The resulting paper [26] was recently submitted to a journal and is currently under refereeing.

2 Objectives and work programme

2.1 Anticipated total duration of the project

36 months

2.2 Objectives

The proposed project has the following three objectives:

A. Find new applications of the product structure theorem for planar graphs.
B. Develop product structure theory further.
C. Push the limits of structural graph theory.

The product structure theorem for planar graphs already found a number of important applications in combinatorics and theoretical computer science in a very short time. It is reasonable to expect that more will be found in the near future. This new tool might indeed be the key to make progress on a number of open problems related to planar graphs. In this proposal, we list some directions which we believe have potential in this respect.

Besides hunting for new applications, we believe it is worth trying to improve and develop the theory further. First, the current bounds are most likely not tight, so there is a room for improvement. Second, it would be very interesting to identify new graph classes of interest that satisfy some form of a product structure theorem. Right now, besides planar graphs, only a handful of examples are known.

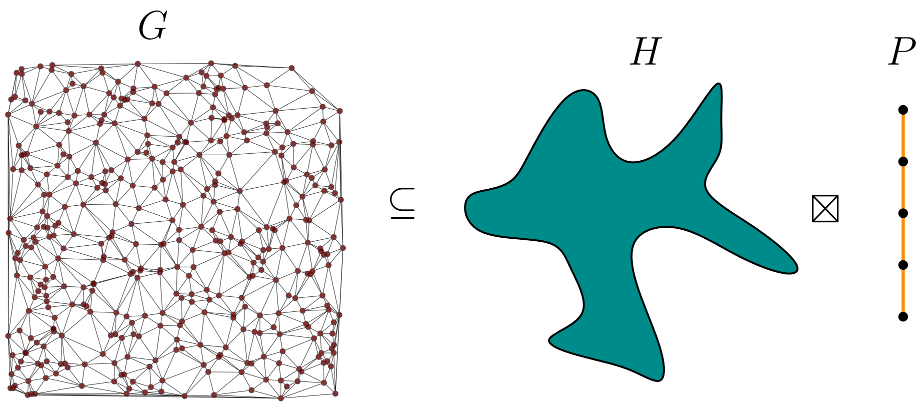
Last but not least, we would like to use the synergy of the three PIs working together, and their respective teams, to have a go at tackling some long standing challenges in structural graph theory. Our focus here will be on obtaining the best possible upper bound in the famous grid minor theorem of Robertson and Seymour, one of the corner stones of graph minor theory with countless applications. This is without doubt an ambitious (and risky) objective.

2.3 Work programme including proposed research methods

First, we recall the statement of the product structure theorem for planar graphs, and give a quick summary of previous applications and generalizations.

Theorem 2 (Product Structure Theorem [24])**.** *Every planar graph is a subgraph of the strong product of a graph of treewidth* 8 *and a path.*

In other words: For every planar graph G, there is a graph H of treewidth 8 and a path P such that G is a subgraph of $H \boxtimes P$, where $\boxtimes$ denotes the strong product. Here is an illustration:



Previous applications

We discovered the product structure theorem when trying to solve a particular problem about planar graphs, namely showing that planar graphs have bounded queue-number. This was the first of a series of applications. In this section we briefly review these applications.

The *queue-number* of a graph is the minimum positive integer c such that its edges can be colored using c colors and its vertices ordered so that no two edges of the same color 'nest'. Two edges nest if they have no common endpoint and the two endpoints of one edge appear strictly between those of the other edge in the ordering. Heath, Leighton, and Rosenberg [36] conjectured in 1992 that the queue-number of every planar graph is bounded from above by an absolute constant. It was well known that graphs of bounded treewidth have bounded queue-number [22, 58]. Using this result in conjunction with the product structure theorem, it is an easy matter to prove the conjecture for planar graphs, see [24].

A vertex coloring of a graph is *nonrepetitive* if there is no path with an even number of vertices such that the sequences of colors on the first and second halves are the same. The *nonrepetitive chromatic number* is the least number of colors in such a coloring. In 1906 Thue [56] showed that there are arbitrarily long squarefree words on an alphabet of size 3, or equivalently, that paths have nonrepetitive chromatic number at most 3. Thue's result lead to the birth of the field of combinatorics on words. In 2002, Alon, Grytczuk, Hałuszczak, and Riordan [2] famously conjectured that this result could be extended to planar graphs, that is, that they have bounded nonrepetitive chromatic number. This became the central problem in this area. With our coauthors we proved this conjecture [19]. Similarly as for queue-numbers, it was well known that graphs of bounded treewidth have bounded nonrepetitive chromatic number [42]. This is then used to show that planar graphs have bounded nonrepetitive chromatic number using the product structure theorem.

In a *labeling scheme*, the task is to assign bitstrings (labels) to the vertices of a graph so that one can determine whether any two vertices are adjacent just by looking at their labels. A classical result of Kannan, Naor, and Rudich [39] constructs such a labeling scheme for n-vertex planar graphs with labels of bit length $4\lceil\log_2 n\rceil$. Shorter bit lengths were achieved in subsequent papers, down to $(2 + o(1))\log_2 n$ in 2007 [31]. Using the product structure theorem, Bonamy, Gavoille, and Pilipczuk [8] improved this to $(4/3 + o(1))\log_2 n$. This is motivated by an old conjecture in the area stating that $(1 + o(1))\log_2 n$ is achievable, which is best possible. As already mentioned, together with our coauthors we recently proved this conjecture [23].

PIs Joret, Micek, and their coauthors [26] recently used the product structure theorem to find (essentially) optimal constructions of universal graphs for planar graphs. For every $n \geq 1$, they construct a graph U_n with only $n^{1+o(1)}$ edges such that U_n contains *every* n-vertex planar graph as a subgraph. In 1982, Babai, Chung, Erdős, Graham, and Spencer constructed such universal graphs with $O(n^{3/2})$ edges, and this remained the best known construction for almost four decades until our work.

The above four applications are the most important ones to date. In each instance, the product structure theorem lead to the solution to well-known problems. The theorem was applied successfully to other problems as well, achieving less dramatic but still significant progress. This includes (1) a near-optimal bound for p-*centered colorings* of planar graphs by PI Micek and his coauthors [17], (2) an asymptotically optimal bound for ℓ-vertex rankings of planar graphs by Bose, Dujmović, Javarsineh, and Morin [9], and (3) a polynomial bound on the 'neighborhood complexity' of planar graphs by PI Joret and his coauthor [38].

Beyond planar graphs

It is natural to wonder if other natural classes of graphs besides planar graphs admit some form of product structure theorem. In our first paper [24], we showed that this is the case for graphs that have a (crossing-free) drawing in a fixed surface:

Theorem 3 (Product Structure Theorem—graphs on surfaces [24]).
Every graph embeddable in a surface of Euler genus g is a subgraph of the strong product of a graph of treewidth $2g+8$ and a path.

A graph J is *apex* if $J-v$ is planar for some vertex v of J. One way to generalize the above result is to consider graphs excluding some fixed apex graph as minor, because this includes graphs of bounded genus: For every surface there is an apex graph that cannot be drawn in it. We showed the following extension:

Theorem 4 (Product Structure Theorem—apex-minor free graphs [24]).
For every apex graph J, there is a constant c_J such that every graph excluding J as minor is a subgraph of the strong product of a graph of treewidth c_J and a path.

A last positive result is due to Dujmović, Morin, and Wood [21] and concerns *k-planar graphs*, graphs that can be drawn in the plane with at most k crossings per edge.

Theorem 5 (Product Structure Theorem—k-planar graphs [21]).
For every $k \geq 1$, every k-planar graph is a subgraph of the strong product of a graph of treewidth $O(k^5)$ and a path.

A motivation for identifying graph classes admitting a product structure theorem is that such classes automatically benefit of the results proved using the product structure, such as all the results mentioned in Section 2.3. (Note that the constants in these results depend on the bound on the treewidth.)

2.3.1 Objective A. Apply product structure theory.

We list here a few open problems that we believe are worth approaching from the perspective of the product structure theorem for planar graphs. The list is not exhaustive, its purpose is to give an idea of potential future applications.

Problem 1: Universal point sets for planar graphs. Say that a set of points in the plane is *n-universal* if every n-vertex planar graph has a straight-line drawing in the plane using points from the set for its vertices. What is the minimum size of an n-universal point set? If we restrict ourselves to realizing *bipartite* planar graphs, then a recent breakthrough [28] provides such point sets of size $O(n)$. However, for arbitrary planar graphs, the best known constructions of n-universal point sets have size $O(n^2)$ [16], which has not been improved since 1990. Yet, no superlinear lower bound is known, the best lower bound is of order $1.235n$ [43]. Improving the asymptotics, i.e., getting a subquadratic upper bound or a superlinear lower bound, is a famous open problem in geometric graph theory. We believe that a $\tilde{O}(n^{3/2})$ upper bound should be provable:

Goal. Show the existence of n-universal point sets of size $\tilde{O}(n^{3/2})$.

(The notation $\tilde{O}$ means 'up to polylog factors'.) Such a result would be a breakthrough. Our reason for considering this problem is the following. In 2015, Fulek and Tóth [30] considered a variant of universal point sets: Instead of considering all n-vertex planar graphs, they only consider those that have treewidth 3. For this variant, they constructed universal point sets of size $\tilde{O}(n^{3/2})$. In the paper proving the product structure theorem [24], we also showed the following variant of the theorem: For every planar G, there is a *planar graph H of treewidth* 3 and a path P, such that G is a subgraph of $H \boxtimes P \boxtimes K_3$ (where K_3 is the triangle). Our goal is thus to lift the Fulek-Tóth result to all planar graphs, using this variant of the product structure theorem.

Problem 2: Straight-line drawings of planar graphs with integer edge lengths. By Fáry's theorem, every planar graph can be drawn with edges represented as straight-line segments. As is well-known, such drawings can be realized with integer coordinates. However this does not imply that all edge lengths are rational, or equivalently after scaling, integer. Harborth [35] conjectured in 1987 that such drawings always exist. Despite much efforts over the years, the conjecture remains widely open. It is known to hold only in a handful of cases, including planar graphs of treewidth 3 [5]. The latter result is especially interesting since, as mentioned above, a variant of the product structure theorem is phrased precisely in terms of planar graphs of treewidth 3. The goal is thus again to lift the result for planar graphs of treewidth 3 to all planar graphs using this theorem.

Goal. Show that planar graphs admit straight-line drawings with integer edge lengths.

Problem 3: Adjacency labelings and universal graphs. A family $\mathcal{G}$ of graphs admits an $f(n)$-*bit adjacency labeling scheme* if there exists a function $A : (\{0,1\}^*)^2 \to \{0,1\}$ such that for every n-vertex graph $G \in \mathcal{G}$ there exists $\ell : V(G) \to \{0,1\}^*$ such that $|\ell(v)| \leq f(n)$ for each vertex v of G and such that, for every two vertices v, w of G,

$$A(\ell(v), \ell(w)) = \begin{cases} 0 & \text{if } vw \notin E(G); \\ 1 & \text{if } vw \in E(G). \end{cases}$$

The best-known labeling scheme for K_t-minor-free graphs has labels of length $(2 + o(1)) \log n$, due to Gavoille and Labourel [31] in 2007.

While PIs Joret, Micek and their coauthors [23] recently obtained adjacency labeling schemes of length $(1 + o(1)) \log n$ for planar graphs, extending this result to K_t-minor free graphs proved to be stumbling point so far. The main difficulty is that the class of K_t-minor-free graphs do not admit a product structure theorem for $t \geq 6$. It seems that all we can do is using the Graph Minor Structure Theorem of Robertson and Seymour to decompose the graphs into pieces that admit a product structure (up to a bounded number of apex vertices). However, handling the decomposition appears to be tricky, and we will probably need to develop a stronger 'weighted' version of the theorem proved in [23] to handle the combination of the pieces.

Goal. Show that, for a each fixed integer $t \geq 1$, K_t-minor-free graphs admit $(1 + o(1)) \log n$ adjacency labeling schemes.

We remark that any bound better than $(2 + o(1)) \log n$ would already be an achievement, as this would require a new technique, the proof method in [31] for K_t-minor free graphs cannot give a better bound than $(2 + o(1)) \log n$.

Problem 4: Weisfeiler-Leman dimension of graphs. The Weisfeiler-Leman dimension of a graph is a measure of its descriptive complexity. It takes its roots in the *Weisfeiler-Leman (WL) algorithm*, a simple heuristic to test for graph isomorphism dating back to the 1960s. (This algorithm is also known as 'color refinement'.) The WL algorithm is a relatively fast algorithm consisting in some simple local computations around each vertex, in order to try and distinguish the two input graphs. If the algorithm succeeds, then the two graphs are non isomorphic. However, the algorithm could fail to distinguish two non-isomorphic graphs given in input. There is a higher dimensional version of the algorithm called the k-*dimensional WL-algorithm*, where k is a positive integer. In this version, local computations are performed for each k-tuple of vertices, instead of each vertex. With larger values of k, the algorithm becomes better at distinguishing non-isomorphic graphs, at the price of an increased running time (which is still polynomial for fixed k).

For a graph G, the *Weisfeiler-Leman (WL) dimension* of G is the smallest positive integer k such that the k-dimensional WL-algorithm distinguishes G from every non-isomorphic graph H. By extension, for a class $\mathcal{C}$ of graphs, the WL-dimension of $\mathcal{C}$ is the smallest positive integer k such that every graph in $\mathcal{C}$ has WL-dimension at most k, or $+\infty$ in case there is no such k.

It is for instance known that the WL-dimension of planar graphs is bounded: Kiefer, Ponomarenko, and Schweitzer [41] proved that planar graphs have WL-dimension at most 3. More generally, WL-dimension is bounded for every proper minor-closed class of graphs, a seminal result in this area due to Grohe [32].

As it turns out, WL-dimension can be characterized in multiple ways, using tools from different areas of theoretical computer science. A key connection is with logic: It was shown [11, 37] that two graphs can be distinguished by the k-dimensional WL-algorithm if and only they can be distinguished by a logic known as the C^{k+1} logic, which is the $(k+1)$-variable fragment of first order logic with counting quantifiers.

Another connection is with combinatorial optimization: Atserias and Maneva [4] showed that the smallest k required to distinguish two non-isomorphic graphs in the k-dimensional WL algorithm is equal to the level of the Sherali-Adams relaxation of a natural formulation of graph isomorphism testing as an integer linear program.

The rich and diverse characterizations of the WL-dimension of a graph G led researchers to consider this invariant as a fundamental measure of the descriptive complexity of G. For instance, the authors of [33] write the following about the k-dimensional WL-algorithm:

> *[...] in view of the wide variety of seemingly unrelated combinatorial, logical, and algebraic characterisations of the algorithm, we are convinced that the structural information the algorithm is able to detect is of fundamental importance.*

This leads us to the question we plan on studying: *Can the k-dimensional WL-algorithm detect graphs having a so-called 'product structure'*? More precisely, we would like to show that such graphs have bounded WL-dimension:

Goal. Given a fixed integer $t \geq 1$, show that the class of graphs G that are subgraphs of $H \boxtimes P$ for some graph H of treewidth t and some path P has WL-dimension bounded by a function of t.

It is known that planar graphs [41] and bounded-genus graphs [32, 33] have bounded WL-dimension, which gives some evidence for the above conjecture. Moreover, the WL-dimension of graphs of treewidth t is known to be at most t [40] (see also [34] for an earlier bound). Thus, to solve our conjecture, it 'only' remains to handle the operations of taking the strong product with a path, and taking subgraphs.

A (positive) solution to the above problem would imply that all graph classes admitting a product structure have bounded WL-dimension. This includes for instance k-planar graphs, for which this is currently unknown as far as we are aware. A second motivation is that it might give a simpler proof that graphs of bounded genus have bounded WL-dimension (likely at the price of a worse bound), as the two existing proofs [32, 33] are rather long and technical.

Problem 5: Extended formulations of spanning tree polytopes. What is the minimum number of facets in a polytope that projects to the spanning tree polytope of an n-vertex graph G? If G is planar, a beautiful proof of Williams [59] gives a $O(n)$ bound, which is best possible. It is an open problem to show a linear bound more generally for graphs embeddable in a fixed surface. The best known bound is $O(n^{3/2})$, proved by PI Joret and his coauthors [29]. We would like to re-prove the linear bound for planar graphs using the product structure, which would then extend directly to the bounded Euler genus case thanks to the extensions mentioned in Section 2.3.

Goal. Given a fixed integer $t \geq 1$, show that for the class of graphs G that are subgraphs of $H \boxtimes P$ for some graph H of treewidth t and some path P, the corresponding spanning tree polytopes admit extended formulations of linear size.

Further problems. Besides the five concrete problems above, there is of course a diverse range of potential applications for the product structure theorems, of which we just mention two examples here. **(1)** What is the smallest k such that every planar graph has 'oriented chromatic number' at most k? It follows from a deep result of Borodin from 1979 that $k \leq 80$ (see [47]), and the bound has never been improved, despite much efforts.[2] **(2)** What is the smallest k such that every planar graph has an 'odd k-coloring'? Introduced just recently [45] as a relative to conflict-free colorings, the best known upper bound is 8 [44]. For both problems, the product structure of planar graphs itself might be too crude a tool to obtain improved bounds on the number of colors. However, we believe that the so-called 'tripod decompositions' used in the proof of the product structure [24] might have potential. Indeed, this proved to be the case recently for an unrelated problem, that of bounding the 'neighborhood complexity' of planar graphs: A first bound was established using the product structure, and then a much better bound was found via a careful use of tripod decompositions [38]. This gives us hope that these decompositions might help improving bounds for the above coloring problems as well.

2.3.2 Objective B. Develop product structure theory.

As already discussed in Section 2.3, product structure is not limited to planar graphs, but constitutes an essential structural property for many graph classes. Let us say that a graph class $\mathcal{G}$ *admits product structure* if there exists a universal constant t such that every graph G in $\mathcal{G}$ is a subgraph of the strong product of a graph of treewidth t and a path. The smallest such t is called the *row treewidth* $\mathrm{rtw}(G)$ of the graph G, respectively $\mathrm{rtw}(\mathcal{G})$ of the graph class $\mathcal{G}$ [10]. So $\mathcal{G}$ admits product structure if and only if $\mathrm{rtw}(\mathcal{G}) = O(1)$.

The known examples of natural graph classes admitting product structure (see Section 2.3) are all very close to planar graphs, and indeed the proofs typically first reduce to the planar case and then apply the product structure theorem for planar graphs. It would be interesting to find classes that are genuinely different from planar graphs. This is very much exploratory. In fact, there has been little research about which graph classes do *not* admit product structure.

Problem 6: Identify necessary conditions and sufficient conditions for a graph class to admit product structure. A first obstruction stems from the treewidth of small radius subgraphs of graphs in $\mathcal{G}$. In fact, if $G \subseteq H \boxtimes P$ with $\mathrm{tw}(H) = t$, then for the k-th neighborhood $N^k(v) = \{u \in V_G : \mathrm{dist}_G(u,v) \leq k\}$ of every vertex v in G we have $N^k(v) \subseteq H \boxtimes P_{2k+1}$, and thus $N^k(v)$ induces a subgraph of G of treewidth at most $\mathrm{tw}(H \boxtimes P_{2k+1}) \leq t(2k+1) = O(k)$, if t is a constant. In other words, a necessary condition for $\mathcal{G}$ to admit product structure is, that the treewidth of subgraphs of $\mathcal{G}$ is linear in their radius; a condition known as *linear local treewidth*. We proved that if $\mathcal{G}$ is minor-closed, then $\mathcal{G}$ having linear

[2] This question was suggested to us by Ross Kang

local treewidth is also sufficient for admitting product structure [24]. However, most recently, Bose et al. [10] proved that for general graph classes it is not.

On one hand, we seek to identify further natural classes of graphs admitting a product structure theorem. We thereby strive for general structural properties that together with linear local treewidth are sufficient to conclude bounded row treewidth. On the other hand, we plan to identify further necessary conditions for product structure, which ideally are efficiently and effectively testable for many graph classes. Here, classes of geometric intersection graphs constitute a promising starting point to approach the above goal, as these are hereditary (closed under taking induced subgraphs) but in general not minor-closed. We plan to mainly consider intersection graphs of specific convex objects in $\mathbb{R}^2$, such as disks or axis-aligned boxes.

In order to avoid arbitrarily large treewidth already in the neighborhood of a vertex (and therefore no linear local treewidth), additional restrictions must be imposed. For example, it is easy to see that unit disk graphs with bounded clique number admit product structure [25], while arbitrary disk graphs with clique number 3 do not even have linear local treewidth, as there can be arbitrarily large grids in the neighborhood of a single vertex. Let us require for a fixed $\alpha \in [0,1]$ that each disk is *α-free*, i.e., has at least an α-proportion of its area disjoint from all other disks. Then for $\alpha = 1$ we obtain the class of planar graphs, which has product structure, while for $\alpha = 0$ we have arbitrarily large cliques and hence no product structure. Varying α strictly between 0 and 1, we can investigate when α-free disk graphs admit product structure and when they admit linear local treewidth. This either leads to a natural non-minor-closed class in which linear local treewidth is sufficient for product structure, or to a new necessary condition for product structure.

Goal. Determine the smallest α_1 and α_2 such that α_1-free disk graphs admit product structure and α_2-free disk graphs have linear local treewidth.

Another natural requirement is to require bounded maximum degree. It seems believable that disk graphs in $\mathbb{R}^2$ with bounded maximum degree have linear local treewidth, while in $\mathbb{R}^3$ for example 3-dimensional grid graphs are contact graphs of unit disks with small maximum degree whose treewidth grows quadratically with the radius. Is it true that disk graphs in $\mathbb{R}^2$ of bounded degree admit product structure? What about axis-aligned boxes, i.e., graphs of boxicity 2? (As interval graphs, i.e., graphs of boxicity 1, are chordal, they admit a trivial product structure of the form $H \boxtimes P_1$ as soon as we bound their clique number.)

Let us remark that this direction of research is very much related to recent efforts led by Maria Chudnovsky and others to identify the induced subgraph obstructions to graphs of bounded treewidth; see for example [1] for the current state of the art.

Problem 7: Approximating product structure. It is well known that computing treewidth is NP-complete [3] but testing $\mathrm{tw}(G) \leq k$ for any fixed k is possible in linear time [7]. A famous open problem is whether treewidth can be approximated to within a constant factor. The current best approximation algorithm achieves an approximation factor of $O(\sqrt{\log n})$ for n-vertex graphs [27]. In a very recent preprint [6], PI Ueckerdt and his coauthors show that computing $\mathrm{rtw}(G)$ is NP-complete, even when deciding $\mathrm{rtw}(G) = 1$ for a graph G with $\mathrm{tw}(G) = 2$. Even approximating the row treewidth is hard; at least when assuming the unproven small set expansion conjecture [60], which would in the same way imply that there is no constant factor approximation for treewidth. On the other hand, treewidth can be approximated to within constant factor for planar graphs [54], while Theorem 1 implies even an additive approximation for row treewidth of planar graphs. Is it possible to use product structure to approximate treewidth? Also, is row treewidth polynomially computable for graphs of bounded radius?

Goal. Study the approximability of row treewidth and its connections with the approximability of treewidth.

Problem 8: Improving product structure. Our goal here is to improve the existing product structure theorems, starting with planar graphs: What is the smallest t such that every planar graph is a subgraph of the strong product of a graph of treewidth t and a path? Our original proof shows $t \leq 8$, and PI Ueckerdt and his coauthors subsequently improved this to $t \leq 6$ [57], but any better bound would need a new idea. In [24] we showed that $t \geq 3$, while treewidth 3 can indeed be achieved if we allow an additional clique factor. Specifically, we show that every planar graph G is a subgraph of $H \boxtimes P \boxtimes K_c$, where $c = 3$, i.e., the complete graph on three vertices. One motivation for reducing c or t is that, in the applications of the product structure theorem (see Section 2.3), the constants depend on the values of t and c.

Goal. Prove or disprove that every planar graph is a subgraph of the strong product of a graph of treewidth 3 and a path.

A further natural question is whether we can impose additional properties of the graph H in the strong product $G \subseteq H \boxtimes P$, possibly if we assume that G meets similar requirements? This direction of research offers many different variations and potential settings. The investigation in each case would lead to a deeper understanding of product structure in general and thus further the development of this powerful theory. Let us mention some concrete examples.

(1) PI Joret, Micek and their coauthors recently showed [20] that we cannot bound the maximum degree of H (while keeping $\mathrm{tw}(H)$ constant), even if G is planar and has maximum degree 5. However, it is an open problem whether every planar graph G of maximum degree Δ is contained in a strong product of the form $H \boxtimes P \boxtimes K_c$ where $\mathrm{tw}(H) = 2$, P is a path, and c is some function of Δ.

(2) Is every planar bipartite graph G a subgraph of $H \boxtimes P$ where H is planar and bipartite and $\mathrm{tw}(H) = O(1)$, or a subgraph of $H \boxtimes P$ where $\mathrm{tw}(H) < 6$?

(3) Is every planar graph G a subgraph of $H \boxslash P$ where $\mathrm{tw}(H) = O(1)$ and $\boxslash$ is the *slanted product* of H and P which for every edge (u, v) of H and (x, y) of P contains the edge between (u, x) and (v, y) but not between (v, x) and (u, y)?

(4) Is every planar n-vertex graph G a subgraph of $H \boxtimes P$ where $\mathrm{tw}(H) = O(1)$ and the length of P is $o(n)$, i.e., sublinear in the number of vertices in G?

2.3.3 Objective C. Push the limits of structural graph theory.

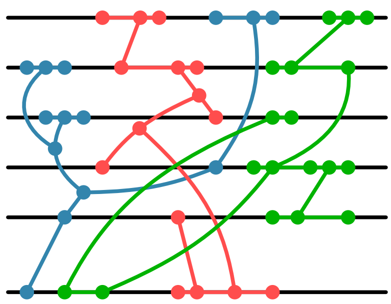
Problem 9: Improve the bound in the Grid Minor Theorem. The Graph Minor Structure Theorem describes the approximate structure of graphs not containing a fixed graph H as minor. In that theorem, H can be any graph. However, if H is planar then the structure theorem becomes much simpler: Robertson and Seymour [50] proved that H-minor-free graphs have bounded treewidth in this case. This is known as the *Grid Minor Theorem*, because it is enough to prove this theorem when H is a grid, since every planar graph is a minor of a large enough grid.

The Grid Minor Theorem has numerous applications in graph theory [48]. It is also of algorithmic and practical importance due to the connection between treewidth and separators: Informally, a graph has treewidth $O(k)$ if and only if all its subgraphs admit $O(k)$-size separators. For this reason, there has been much interest in getting the best-possible bound on the treewidth of graphs with no $k \times k$ grid minor. In sharp contrast with the general Graph Minor Structure Theorem, here very good bounds are known. Most notably, a breakthrough result of Chekuri and Chuzhoy [14] gives a bound that is polynomial in k. Currently, the best known bound is $O(k^9 \operatorname{polylog}(k))$, proved by Chuzhoy and Tan [15] in 2019. Robertson, Seymour, and Thomas [53] conjectured in 1994 that the best possible bound is $O(k^2 \log k)$. This bound is attained by random graphs, which often provide tight examples in structural graph theory.

Goal. Prove a tight upper bound on the treewidth of graphs with no $k \times k$ grid minor.

We are eager to pursue this ambitious goal now, for the following reason. A well-known corollary of the Grid Minor Theorem, proved by Robertson and Seymour in their original paper [50], is the following approximate min-max relation: For every fixed planar graph H there exists a function $f(k)$ such that every graph G either has k vertex disjoint copies of H as a minor, or there is a set X of at most $f(k)$ vertices such that $G - X$ has no H-minor. As a forerunner of their polynomial bound for the Grid Minor Theorem, Chekuri and Chuzhoy [13] established a near-optimal bound for the latter result, a bound of $f(k) = O(k \log^c k)$ for some absolute constant c. It was conjectured long ago that the right bound is $O(k \log k)$, i.e. we can take $c = 1$. This was eventually proved by PI Joret and his coauthors [12] recently. The proof approach is different from [13] and introduces some new ideas that could be helpful in attacking the above problem.

Here are some further details on the proposed approach. The goal is to establish a tight bound on the treewidth of graphs with no $k \times k$ grid minor, which is conjectured to be $O(k^2 \log k)$. In an attempt at getting a polynomial bound for the Grid Minor Theorem that predates the breakthrough of Chekuri and Chuzhoy [14], Reed and Wood [49] introduced a generalization of grid minors called *grid-like minors*: These are composed of a collection of paths in the graph whose intersection graphs are bipartite and contain a large clique minor. While these grid-like minors turned out to be not structured enough to deduce the desired polynomial bound for the Grid Minor Theorem, they provided inspiration for the concept of *orchards* introduced by PI Joret and his coauthors [12]. This is another generalization of grid minors. An orchard consists of a collection of pairwise vertex-disjoint paths (*horizontal paths*) and a collection of vertex-disjoint trees (*vertical trees*) such that every vertical tree intersects every horizontal path in a non-empty connected subgraph.



An orchard. Horizontal paths are depicted in black and vertical trees in color.

While orchards and grid-like minors have common features (note that the intersection graph of the horizontal paths and vertical trees of an orchard is a complete bipartite graph), in general they are incomparable objects. Orchards, and in particular a special type of orchards called *tame orchards*, are a key concept in the recent proof by PI Joret and his coauthors [12] of an optimal bound for the so-called *Erdős-Pósa property* of planar minors. The latter theorem was originally derived as a corollary of the Grid Minor Theorem by Robertson and Seymour [50]. It is thus natural to expect that orchards, and other techniques from [12], could be helpful in improving the existing bounds for the Grid Minor Theorem itself. This will be the starting point of our investigations.

We remark that, while our ultimate goal is to achieve a tight upper bound on the treewidth of graphs with no $k \times k$ grid minor, any improvement on the current best known bound of $O(k^9 \operatorname{polylog}(k))$ [15] would already be a significant achievement.

2.4 Handling of research data

N/A

2.5 Relevance of sex, gender and/or diversity

N/A

3 Project- and subject-related list of publications

Works cited from sections 1 and 2, both by the applicant(s) and by third parties. Please include DOI/URL if available. ***A maximum of ten*** *of your own works cited may be* ***highlighted****; font at least Arial 9 pt.*

[1] Tara Abrishami, Bogdan Alecu, Maria Chudnovsky, Sepehr Hajebi, and Sophie Spirkl. *Induced subgraphs and tree decompositions VIII. Excluding a forest in (theta, prism)-free graphs*. 2023. DOI: 10.48550/arXiv.2301.02138.

[2] Noga Alon, Jarosław Grytczuk, Mariusz Hałuszczak, and Oliver Riordan. "Nonrepetitive colorings of graphs." In: *Random Structures Algorithms* 21.3-4 (2002), pp. 336–346. DOI: 10.1002/rsa.10057.

[3] Stefan Arnborg, Derek G. Corneil, and Andrzej Proskurowski. "Complexity of Finding Embeddings in a k-Tree." In: *SIAM Journal on Algebraic Discrete Methods* 8.2 (1987), pp. 277–284. DOI: 10.1137/0608024.

[4] Albert Atserias and Elitza N. Maneva. "Sherali-Adams Relaxations and Indistinguishability in Counting Logics." In: *SIAM Journal on Computing* 42.1 (2013), pp. 112–137. DOI: 10.1137/120867834.

[5] Thomas G. Berry. "Points at rational distance from the vertices of a triangle." In: *Acta Arithmetica* 62.4 (1992), pp. 391–398. DOI: 10.4064/aa-62-4-391-398.

[6] Therese Biedl, David Eppstein, and Torsten Ueckerdt. *On the complexity of embedding in graph products*. 2023. DOI: 10.48550/arXiv.2303.17028.

[7] Hans L. Bodlaender. "A Linear-Time Algorithm for Finding Tree-Decompositions of Small Treewidth." In: *SIAM Journal on Computing* 25.6 (1996), pp. 1305–1317. DOI: 10.1137/S0097539793251219.

[8] Marthe Bonamy, Cyril Gavoille, and Michał Pilipczuk. "Shorter Labeling Schemes for Planar Graphs." In: *SIAM Journal on Discrete Mathematics* 36.3 (2022), pp. 2082–2099. DOI: 10.1137/20M1330464.

[9] Prosenjit Bose, Vida Dujmović, Mehrnoosh Javarsineh, and Pat Morin. *Asymptotically Optimal Vertex Ranking of Planar Graphs*. 2022. DOI: 10.48550/arXiv.2007.06455.

[10] Prosenjit Bose, Vida Dujmović, Mehrnoosh Javarsineh, Pat Morin, and David R. Wood. "Separating layered treewidth and row treewidth." In: *Discrete Mathematics & Theoretical Computer Science* vol. 24, no. 1 (May 2022). DOI: 10.46298/dmtcs.7458.

[11] Jin-yi Cai, Martin Fürer, and Neil Immerman. "An optimal lower bound on the number of variables for graph identification." In: *Combinatorica* 12.4 (1992), pp. 389–410. DOI: 10.1007/BF01305232.

[12] **Wouter Cames van Batenburg, Tony Huynh, Gwenaël Joret, and Jean-Florent Raymond. "A tight Erdős-Pósa function for planar minors." In: *Advances in Combinatorics* (2019), P2, 33 pp. DOI: 10.19086/aic.10807.**

[13] Chandra Chekuri and Julia Chuzhoy. "Large-treewidth graph decompositions and applications." In: *Proceedings of the 45th annual ACM Symposium on Theory of Computing (STOC 2013)*. ACM. 2013, pp. 291–300. DOI: 10.1145/2488608.2488645.

[14] Chandra Chekuri and Julia Chuzhoy. "Polynomial bounds for the grid-minor theorem." In: *Journal of the ACM* 63.5 (2016), 40:1–40:65. DOI: 10.1145/2820609.

[15] Julia Chuzhoy and Zihan Tan. "Towards Tight(er) Bounds for the Excluded Grid Theorem." In: *Proceedings of the Thirtieth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA 2019)*. ACM, pp. 1445–1464. DOI: 10.1137/1.9781611975482.88.

[16] Hubert De Fraysseix, János Pach, and Richard Pollack. "How to draw a planar graph on a grid." In: *Combinatorica* 10 (1990), pp. 41–51. DOI: 10.1007/BF02122694.

[17] **Michał Dębski, Stefan Felsner, Piotr Micek, and Felix Schröder. "Improved bounds for centered colorings." In: *Advances in Combinatorics* 2021 (2021). No 8, p. 28. DOI: 10.19086/aic.27351.**

[18] Reinhard Diestel. *Graph theory*. Fourth. Vol. 173. Graduate Texts in Mathematics. Springer, Heidelberg, 2010, pp. xviii+437. ISBN: 978-3-642-14278-9. DOI: 10.1007/978-3-642-14279-6.

[19] **Vida Dujmović, Louis Esperet, Gwenaël Joret, Bartosz Walczak, and David R. Wood. "Planar graphs have bounded nonrepetitive chromatic number." In: *Advances in Combinatorics* (2020), P5, 11 pp. DOI: 10.19086/aic.12100.**

[20] Vida Dujmović, Gwenaël Joret, Piotr Micek, Pat Morin, and David R. Wood. *Bounded-Degree Planar Graphs Do Not Have Bounded-Degree Product Structure*. 2022. DOI: 10.48550/arXiv.2212.02388.

[21] Vida Dujmović, Pat Morin, and David R. Wood. *Graph product structure for non-minor-closed classes*. 2022. DOI: 10.48550/arXiv.1907.05168.

[22] Vida Dujmović, Pat Morin, and David R. Wood. "Layout of Graphs with Bounded Tree-Width." In: *SIAM Journal on Computing* 34.3 (2005), pp. 553–579. DOI: 10.1137/S0097539702416141.

[23] **Vida Dujmović et al. "Adjacency Labelling for Planar Graphs (and Beyond)." In: *Journal of the ACM* 68.6 (2021), 42:1–42:33. DOI: 10.1145/3477542.**

[24] **Vida Dujmović et al. "Planar Graphs Have Bounded Queue-Number." In: *Journal of the ACM* 67.4 (2020), 22:1–22:38. DOI: 10.1145/3385731.**

[25] Zdeněk Dvořák, Tony Huynh, Gwenaël Joret, Chun-Hung Liu, and David R. Wood. "Notes on graph product structure theory." In: *2019-20 MATRIX Annals*. Ed. by D.R. Wood, J. de Gier, C.E. Praeger, and T. Tao. Vol. 4. MATRIX Book Series. Springer, 2021, pp. 513–533. DOI: 10.1007/978-3-030-62497-2_32.

[26] Louis Esperet, Gwenaël Joret, and Pat Morin. *Sparse universal graphs for planarity*. 2022. DOI: 10.48550/arXiv.2010.05779.

[27] Uriel Feige, MohammadTaghi Hajiaghayi, and James R. Lee. "Improved approximation algorithms for minimum weight vertex separators." In: *SIAM Journal on Computing* 38.2 (2008), pp. 629–657. DOI: 10.1137/05064299X.

[28] Stefan Felsner, Hendrik Schrezenmaier, Felix Schröder, and Raphael Steiner. *Linear Size Universal Point Sets for Classes of Planar Graphs*. 2023. DOI: 10.48550/arXiv.2303.00109.

[29] **Samuel Fiorini, Tony Huynh, Gwenaël Joret, and Kanstantsin Pashkovich. "Smaller extended formulations for the spanning tree polytope of bounded-genus graphs." In: *Discrete and Computational Geometry* 57.3 (2017), pp. 757–761. DOI: 10.1007/s00454-016-9852-9.**

[30] Radoslav Fulek and Csaba D. Tóth. "Universal point sets for planar three-trees." In: *Journal of Discrete Algorithms* 30 (2015), pp. 101–112. DOI: 10.1016/j.jda.2014.12.005.

[31] Cyril Gavoille and Arnaud Labourel. "Shorter Implicit Representation for Planar Graphs and Bounded Treewidth Graphs." In: *Algorithms - ESA 2007, 15th Annual European Symposium, Eilat, Israel, October 8-10, 2007, Proceedings*. Ed. by Lars Arge, Michael Hoffmann, and Emo Welzl. Vol. 4698. Lecture Notes in Computer Science. Springer, 2007, pp. 582–593. DOI: 10.1007/978-3-540-75520-3_52.

[32] Martin Grohe. "Fixed-point definability and polynomial time on graphs with excluded minors." In: *Journal of the ACM* 59.5 (2012), 27:1–27:64. DOI: 10.1145/2371656.2371662.

[33] Martin Grohe and Sandra Kiefer. "A Linear Upper Bound on the Weisfeiler-Leman Dimension of Graphs of Bounded Genus." In: *46th International Colloquium on Automata, Languages, and Programming, ICALP 2019, July 9-12, 2019, Patras, Greece*. Ed. by Christel Baier, Ioannis Chatzigiannakis, Paola Flocchini, and Stefano Leonardi. Vol. 132. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2019, 117:1–117:15. DOI: 10.4230/LIPIcs.ICALP.2019.117.

[34] Martin Grohe and Julian Mariño. "Definability and Descriptive Complexity on Databases of Bounded Tree-Width." In: *Database Theory - ICDT '99, 7th International Conference, Jerusalem, Israel, January 10-12, 1999, Proceedings*. Ed. by Catriel Beeri and Peter Buneman. Vol. 1540. Lecture Notes in Computer Science. Springer, 1999, pp. 70–82. DOI: 10.1007/3-540-49257-7_6.

[35] Heiko Harborth, Arnfried Kemnitz, Meinhard Möller, and Andrea Süssenbach. "Ganzzahlige planare Darstellungen der platonischen Körper. (Integral planar representations of the platonic polyhedra)." German. In: *Elem. Math.* 42.5 (1987), pp. 118–122.

[36] Lenwood S. Heath, F. Thomson Leighton, and Arnold L. Rosenberg. "Comparing queues and stacks as mechanisms for laying out graphs." In: *SIAM Journal on Discrete Mathematics* 5.3 (1992), pp. 398–412. DOI: 10.1137/0405031.

[37] Neil Immerman and Eric Lander. "Describing Graphs: A First-Order Approach to Graph Canonization." In: *Complexity Theory Retrospective: In Honor of Juris Hartmanis on the Occasion of His Sixtieth Birthday, July 5, 1988*. Ed. by Alan L. Selman. New York, NY: Springer New York, 1990, pp. 59–81. ISBN: 978-1-4612-4478-3. DOI: 10.1007/978-1-4612-4478-3_5.

[38] Gwenaël Joret and Clément Rambaud. *Neighborhood complexity of planar graphs*. 2023. DOI: 10.48550/arXiv.2302.12633.

[39] Sampath Kannan, Moni Naor, and Steven Rudich. "Implicit representation of graphs." In: *SIAM Journal on Discrete Mathematics* 5.4 (1992), pp. 596–603. DOI: 10.1137/0405049.

[40] Sandra Kiefer and Daniel Neuen. "The Power of the Weisfeiler-Leman Algorithm to Decompose Graphs." In: *SIAM Journal on Discrete Mathematics* 36.1 (2022), pp. 252–298. DOI: 10.1137/20m1314987.

[41] Sandra Kiefer, Ilia Ponomarenko, and Pascal Schweitzer. "The Weisfeiler-Leman Dimension of Planar Graphs Is at Most 3." In: *Journal of the ACM* 66.6 (2019), 44:1–44:31. DOI: 10.1145/3333003.

[42] André Kündgen and Michael J. Pelsmajer. "Nonrepetitive colorings of graphs of bounded tree-width." In: *Discrete Mathematics* 308.19 (2008), pp. 4473–4478. DOI: 10.1016/j.disc.2007.08.043.

[43] Maciej Kurowski. "A 1.235 lower bound on the number of points needed to draw all n-vertex planar graphs." In: *Information Processing Letters* 92.2 (2004), pp. 95–98. DOI: 10.1016/j.ipl.2004.06.009.

[44] Jan Petr and Julien Portier. *The odd chromatic number of a planar graph is at most 8*. 2023. DOI: 10.1007/s00373-023-02617-z.

[45] Mirko Petruševski and Riste Škrekovski. *Colorings with neighborhood parity condition*. 2021. DOI: 10.48550/arXiv.2112.13710.

[46] Michał Pilipczuk and Sebastian Siebertz. "Polynomial bounds for centered colorings on proper minor-closed graph classes." In: *Journal of Combinatorial Theory, Series B* 151 (2021), pp. 111–147. DOI: 10.1016/j.jctb.2021.06.002.

[47] André Raspaud and Eric Sopena. "Good and semi-strong colorings of oriented planar graphs." In: *Information Processing Letters* 51.4 (1994), pp. 171–174. DOI: 10.1016/0020-0190(94)00088-3.

[48] Bruce A. Reed. "Tree Width and Tangles: A New Connectivity Measure and Some Applications." In: *Surveys in Combinatorics, 1997*. Ed. by R. A.Editor Bailey. London Mathematical Society Lecture Note Series. Cambridge University Press, 1997, pp. 87–162. DOI: 10.1017/CBO9780511662119.006.

[49] Bruce A. Reed and David R. Wood. "Polynomial treewidth forces a large grid-like-minor." In: *European Journal of Combinatorics* 33.3 (2012), pp. 374–379. DOI: 10.1016/j.ejc.2011.09.004.

[50] Neil Robertson and Paul Seymour. "Graph minors. V. Excluding a planar graph." In: *Journal of Combinatorial Theory, Series B* 41.1 (1986), pp. 92–114. DOI: 10.1016/0095-8956(86)90030-4.

[51] Neil Robertson and Paul Seymour. "Graph minors. XVI. Excluding a non-planar graph." In: *Journal of Combinatorial Theory, Series B* 89.1 (2003), pp. 43–76. DOI: 10.1016/S0095-8956(03)00042-X.

[52] Neil Robertson and Paul Seymour. "Graph Minors. XX. Wagner's conjecture." In: *Journal of Combinatorial Theory, Series B* 92.2 (2004), pp. 325–357. DOI: 10.1016/j.jctb.2004.08.001.

[53] Neil Robertson, Paul Seymour, and Robin Thomas. "Quickly excluding a planar graph." In: *Journal of Combinatorial Theory, Series B* 62.2 (1994), pp. 323–348. DOI: 10.1006/jctb.1994.1073.

[54] Paul Seymour and Robin Thomas. "Call routing and the ratcatcher." In: *Combinatorica* 14 (1994), pp. 217–241. DOI: 10.1007/BF01215352.

[55] Endre Szemerédi. "Regular partitions of graphs." In: *Problèmes combinatoires et théorie des graphes*. Vol. 260. Colloq. Internat. CNRS. CNRS, Paris, 1978, pp. 399–401.

[56] Axel Thue. "Über unendliche Zeichenreihen." In: *Norske Vid. Selsk. Skr. I. Mat. Nat. Kl. Christiania* 7 (1906), pp. 1–22.

[57] **Torsten Ueckerdt, David R. Wood, and Wendy Yi. "An improved planar graph product structure theorem." In: *Electronic Journal of Combinatorics* 29 (2 2022). DOI:** 10.37236/10614**.**

[58] Veit Wiechert. "On the queue-number of graphs with bounded tree-width." In: *Electronic Journal of Combinatorics* 24.1 (2017), p. 1.65. DOI: 10.37236/6429.

[59] Justin C. Williams. "A linear-size zero-one programming model for the minimum spanning tree problem in planar graphs." In: *Networks* 39.1 (2002), pp. 53–60. DOI: 10.1002/net.10010.

[60] Yu Wu, Per Austrin, Toniann Pitassi, and David Liu. "Inapproximability of treewidth, one-shot pebbling, and related layout problems." In: *J. Artificial Intelligence Research* 49 (2014), 569–600. DOI: 10.1613/jair.4030.

4 Supplementary information on the research context

Section 4 et seq. must not exceed 8 pages.

4.1 Ethical and/or legal aspects of the project

4.1.1 General ethical aspects

N/A

4.1.2 Description of proposed investigations involving humans, human materials or identifiable data

N/A

4.1.3 Descriptions of proposed investigations involving experiments on animals

N/A

4.1.4 Descriptions of projects involving genetic resources (or associated traditional knowledge) from a foreign country

N/A

4.1.5 Explanations regarding any possible safety-related aspects ("Dual Use Research of Concern; foreign trade law)

N/A

4.2 Employment status information

For each applicant, state the last name, first name, and employment status (including duration of contract and funding body, if on a fixed-term contract).

Joret, Gwenaël:

Associate Professor (*Chargé de Cours*), Université libre de Bruxelles, permanent position

Micek, Piotr:

Professor (*Profesor UJ*), Jagiellonian University, permanent position

Ueckerdt, Torsten:

Senior Researcher, Karlsruhe Institute of Technology, contract until October 2024
→ admitted to DFG Heisenberg Programme in March 2023

4.3 First-time proposal data

Only if applicable: Last name, first name of first-time applicant

N/A

4.4 Composition of the project group

List only those individuals who will work on the project but will not be paid out of the project funds. State each person's name, academic title, employment status, and type of funding.

N/A

4.5 Researchers in Germany with whom you have agreed to cooperate on this project

N/A

4.6 Researchers abroad with whom you have agreed to cooperate on this project

Cross-border cooperation in a weave lead-agency process:

Gwenaël Joret (Université libre de Bruxelles, Belgium) • Piotr Micek (Jagiellonian University, Krakow, Poland)

General international research cooperation:

Vida Dujmović (University of Ottawa) • Louis Esperet (G-SCOP Laboratory, Grenoble) • Pat Morin (Carleton University) • David R. Wood (Monash University)

4.7 Researchers with whom you have collaborated scientifically within the past three years

This information will help avoid potential conflicts of interest.

Therese Biedl • Steven Chaplick • Vida Dujmović • Zdeněk Dvořák • David Eppstein • Louis Esperet • Stefan Felsner • Cyril Gavoille • Daniel Gonçalves • Michael Kaufmann • Balázs Keszegh • Kolja Knauer • Stephen Kobourov • Pat Morin • Sergey Norin • János Pach • Dömötör Pálvölgyi • Marcin Pilipczuk • Michał Pilipczuk • Géza Tóth • Pavel Valtr • William T. Trotter • Bartosz Walczak • David R. Wood

4.8 Project-relevant cooperation with commercial enterprises

If applicable, please note the EU guidelines on state aid or contact your research institution in this regard.

N/A

4.9 Project-relevant participation in commercial enterprises

Information on connections between the project and the production branch of the enterprise

N/A

4.10 Scientific equipment

List larger instruments that will be available to you for the project. These may include large computer facilities if computing capacity will be needed.

N/A

4.11 Other submissions

List any funding proposals for this project and/or major instrumentation previously submitted to a third party.

N/A

4.12 Other information

Please use this section for any additional information you feel is relevant which has not been provided elsewhere.

N/A

5 Requested modules/funds

Explain each item for each applicant (stating last name, first name).

5.1 Basic Module

Ueckerdt, Torsten:
Basic Module

5.1.1 Funding for Staff

Joret, Gwenaël:
one postdoctoral researcher, 100%, for 3 years. Total: € 157 500.

Micek, Piotr:
additional salary for PI, for 3 years. Total: € 22 860
one postdoctoral researcher, for 2 years. Total: € 59 267
scholarship for PhD student, for 2 years. Total: € 25 400
scholarship for Phd/MSc student, for 3 years. Total: € 22 860
In total: € 130 387

Ueckerdt, Torsten:
one postdoctoral researcher, 100%, for 3 years. Total: € 240 300.

Requirements for postdoctoral positions: PhD degree in mathematics or computer science. Strong profile in theoretical computer science. Expertise in structural graph theory and graph theory in general. Preferably, experience with the product structure.

Requirements for student positions: Excellent track record, interest in research and open mind.

5.1.2 Direct Project Costs

5.1.2.1 Equipment up to € 10,000, Software and Consumables

Micek, Piotr:
Laptop or tablet: € 2 117.

Almost all paperwork is done on a laptop these days. A reliable laptop (or tablet) is necessary to prepare manuscripts, presentations, and to participate in various planned and ad-hoc research meetings.

5.1.2.2 Travel Expenses

Joret, Gwenaël:
Funds to cover transportation and conference fees: € 5 000 per year. Total: € 15 000.

Micek, Piotr:
Funds to cover transportation and conference fees: € 5 000 per year. Total: € 15 000

Ueckerdt, Torsten:
Funds to cover transportation and conference fees: € 5 000 per year. Total: € 15 000.

We plan a participation of each project member in one/two conferences per year (on average) with the purpose of presenting the results of the project and (in the case of students) gaining the knowledge of current research trends. This is to include important conferences in combinatorics and theoretical computer science (e.g. SODA, FOCS and STOC).

For the research visits the priority will always be to maintain high-quality research collaboration. Additionally, regular travel between the participating institutions is key to the success of this international collaboration.

This includes flight tickets, accommodation, per diems, and fees. We estimate an average cost of one such business trip to be € 1 500 for Europe and € 2 500 outside Europe. We allocate € 5 000 for each year of the project. This should cover about 2-4 such trips per year.

5.1.2.3 Visiting Researchers (excluding Mercator Fellows)

Micek, Piotr:
Funds to host visiting researchers: € 3 000 per year. Total: € 9 000.

Ueckerdt, Torsten:
Funds to host visiting researchers: € 4 000 per year. Total: € 12 000.

Hosting research visitors, colleagues, and guests is equally important to travelling. And it is much cheaper. Among the intended research guests are: Zdeněk Dvořák (Charles University), Stefan Felsner (TU Berlin), Louis Esperet (CNRS Grenoble), Patrice Ossona de Mendez (CNRS Paris), David Wood (Monash, Melbourne), and many more. Estimated cost of hosting one-week research visit in Kraków is € 900. For Karlsruhe it is € 1 200.

5.1.2.4 Expenses for Laboratory Animals

N/A

5.1.2.5 Other Costs

Micek, Piotr:
indirect costs: € 31 301

The default amount of indirect costs in research projects funded by NCN at Jagiellonian University is 20%.

5.1.2.6 Project-related Publication Expenses

Micek, Piotr:
cost of Open Access: € 3 130

An obligatory item in research projects funded by NCN: 2% of the direct costs supporting the Open Access actions.

5.1.3 Instrumentation

5.1.3.1 Equipment exceeding € 10,000

N/A

5.1.3.2 Major Instrumentation exceeding € 50,000

N/A

5.2 Module Temporary Position for Principal Investigator

N/A

5.3 Module Replacement Funding

N/A

5.4 Module Temporary Clinician Substitute

N/A

5.5 Module Mercator Fellows

N/A

5.6 Module Workshop Funding

N/A

5.7 Module Public Relations Funding

N/A

5.8 Module Standard Allowance for Gender Equality Measures

Please detail what measures are planned to promote diversity and equal opportunities. If you are submitting your proposal for an individual research grant within a ***network****, note that this standard allowance may only be applied for within the coordination project. The coordination project must combine all such requests in its calculation.*

N/A